Natural convection above fires

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The turbulent natural convection above fires in a dry calm atmosphere with a constant lapse rate has been the subject of several recent investigations (see references). The present paper presents solution curves from which the natural convection may be computed over a fire of arbitrary size in an atmosphere with arbitrary lapse-rate variation. The independent parameters of fire size, energy release rate (buoyancy), momentum release rate and atmospheric lapse rate are given over the expected range of values. The arbitrary variation of lapse rate is thus calculable as piecewise constant.

1. Introduction

The hot gasses produced by a fire, being lighter than the ambient air, experience an upward buoyant force. Thus a column of rising hot gas is formed. If the Reynolds number of this column is low enough, the flow will remain laminar and additional air will be mixed in and carried upward through molecular processes only. If the Reynolds number is high the gas column becomes turbulent and the mixing of additional air is greatly increased. Although the precise conditions required for a completely laminar convection column are not known (at least to the authors), recent experiments indicate that for open-pan fires the hot gas column is largely turbulent for all pan sizes larger than a few inches. The turbulent flow may take a distance of a pan diameter or two to become established but all further convection column flow is fully turbulent. For this reason the present work is confined to a treatment of the turbulent convection column.

As the column rises it cools due to both expansion and the mixing of atmospheric air. Thus in an atmosphere with a stable lapse rate, the column slows its rise, finally stops, and then falls back to an equilibrium level. For an atmosphere with an unstable lapse rate, any disturbance will grow to produce a large eddy. The rising column acts as such a disturbance, is augmented as it rises, and thus localizes one of the points of overturning. For an arbitrary lapse-rate variation, the solution to the convection problem would have to be carried out for each case from the beginning, since no sufficiently simple solution has yet been found. However, if the atmosphere is considered as of piecewise constant lapse rate, and the solution is given for arbitrary fire size b^0, velocity u^0 and buoyancy $\Delta\gamma^0$, then it is possible to compute the entire column by using the final b, u, $\Delta\gamma$ values from one section as the starting values of the next.

There are several methods of treatment of the turbulent flow in the rising

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column. Schmidt (1941), the first to examine this problem, neglected vertical diffusion and used both the Prandtl momentum transfer and Taylor vorticity theories. The mixing length was assumed equal to the plume width. Later Rouse, Yih & Humpheries (1952) independently made a similar investigation and showed close agreement with experimental results. Priestly & Ball (1955) gave a solution for the movement of smoke from chimneys. They assume that the plume spreads linearly with height from the virtual source. Indirectly this is equivalent to a mechanism of turbulence.

A simpler transfer mechanism was assumed by Morton, Taylor & Turner (1956). Their solutions also agree well with experiment, thus justifying their simpler assumption. Morton (1957) later included the effect of moisture on the rising column. He also more recently (1959) extended his previous work to the mathematical discussion of finite size, positive and negative buoyancy sources in a stable atmosphere.

The effect of moisture is worthy of note since its condensation is sometimes observed over fires and such condensation alters, considerably, the convection column rise.

In the present work, the convection problem is reduced to the smallest possible number of parameters that still permits calculation of a very general case. No assumption of virtual source is made. No additional assumptions are made, but a new set of dimensionless variables is required by the desire to compute a general atmosphere in a piecewise constant manner.

2. Fundamental equations

Consider a cylindrically symmetrical convection column. Take the $\bar{x}$-axis vertically upward, and r radial. The corresponding velocity components u, v, the density ρ, the pressure p and all other fluid properties are assumed to be local mean values. Thus turbulent components are averaged out or included in other terms as shear stress τ or heat flux q. Finally, we assume that the vertical pressure distribution is given by the usual hydrostatic approximation,

$$p = p_0 - \gamma_\infty \bar{x}, \tag{1}$$

γ_∞ being the specific weight of the fluid outside the plume at infinity, and p_0 the standard pressure. The equations of conservation of mass, momentum and energy are then

$$\frac{\partial \gamma r u}{\partial \bar{x}} + \frac{\partial \gamma r v}{\partial r} = 0, \tag{2}$$

$$ru\frac{\partial u}{\partial \bar{x}} + rv\frac{\partial u}{\partial r} = \frac{r(\gamma_\infty - \gamma)}{\rho} + \frac{1}{\rho}\frac{\partial r\tau}{\partial r}, \tag{3}$$

$$r\left(u\frac{\partial h}{\partial \bar{x}} + v\frac{\partial h}{\partial r} + ug\right) = -\frac{1}{\rho}\frac{\partial rq}{\partial r}, \tag{4}$$

in which γ is the specific weight of the fluid in the plume, h is the enthalpy per unit volume, g is the acceleration due to gravity, and τ and q are the vertical shear stress and radial component of heat flux, being given, for linear turbulence, by $r\tau = -\rho\overline{v'u'}$ and $rq = \rho c_p \overline{v'T'}$, respectively. u', v' and T' are the fluctuating components of u, v and T, and c_p the average specific heat of air at constant pressure. While deriving the equation (4), the dissipation function and vertical heat flux have been neglected.

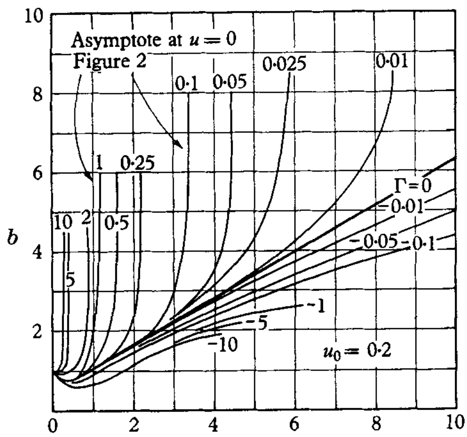

FIGURE 1. The variation of plume width b with height x and atmospheric lapse Γ. Parameter u_0 is initial plume velocity.

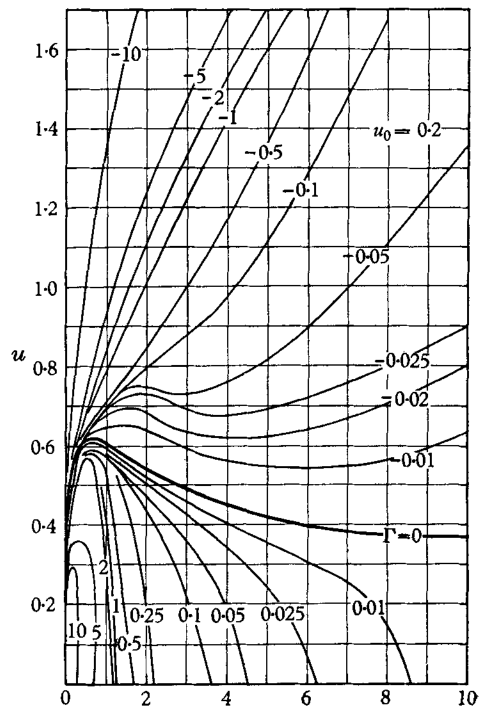

FIGURE 2. The variation of plume velocity u with height x and atmospheric lapse rate Γ. Parameter u_0 is initial plume velocity.

The major effects of the decrease of atmospheric pressure with height are taken into account in a simple way by introducing the potential temperature T_0 and specific weight γ_0 defined by

$$T_0 = \left(\frac{p_0}{p}\right)^{(k-1)/k} T, \quad \gamma_0 = \left(\frac{p_0}{p}\right)^{1/k} \gamma, \tag{5}$$

T being the absolute temperature and k the ratio of specific heats. After some manipulation the conservation equations become

$$\frac{\partial ru}{\partial \bar{x}} + \frac{\partial rv}{\partial r} = 0, \tag{6}$$

$$ru\frac{\partial u}{\partial \bar{x}} + rv\frac{\partial u}{\partial r} = \frac{rg\Delta\gamma_0}{\gamma_0} + \frac{1}{\rho_0}\frac{\partial r\tau_0}{\partial r}, \tag{7}$$

$$ru\frac{\partial}{\partial \bar{x}}\left(\frac{\Delta\gamma_0}{\gamma_0}\right) + rv\frac{\partial}{\partial r}\left(\frac{\Delta\gamma_0}{\gamma_0}\right) + ru\frac{d\ln T_{\infty 0}}{d\bar{x}} = \frac{1}{c_p T_0 \rho_0}\frac{\partial r q_0}{\partial r}, \tag{8}$$

where density variations are neglected in the continuity equation and a term involving variations of potential density is omitted from the energy equation (see Appendix for details). The quantities with subscript 0 refer to the potential variables.

By integration of each of (6), (7), (8) from $r = 0$ to $r = \infty$, mean values of various quantities are defined. This is not a unique set of definitions but has some

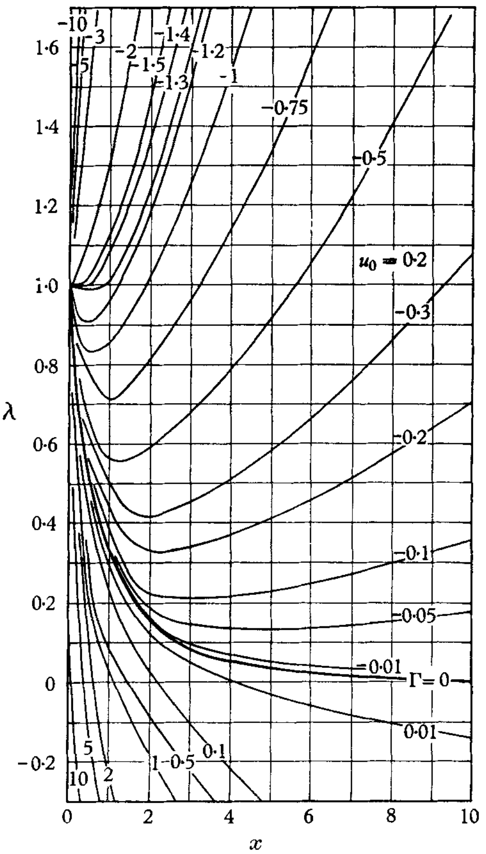

FIGURE 3. The variation of plume buoyancy λ with height x and atmospheric lapse rate Γ. Parameter u_0 is initial plume velocity.

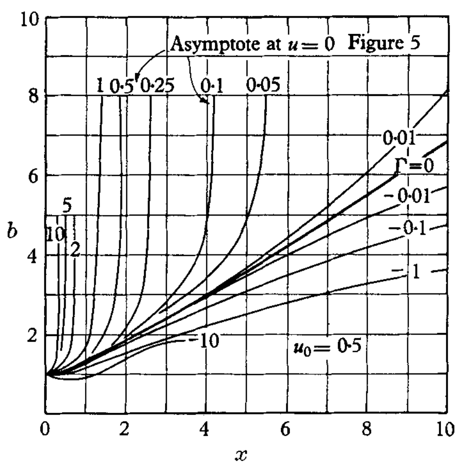

FIGURE 4. The variation of plume width b with height x and atmospheric lapse rate Γ. Parameter u_0 is initial plume velocity.

simple properties as noted below. The mean values may be looked upon as defining the equivalent top-hat profiles. Thus we obtain the mean velocity

$$\bar{u} = \frac{\int_0^\infty ru^2 dr}{\int_0^\infty ru\, dr}, \tag{9}$$

the mean width b, given by

$$\bar{b}^2 = \frac{2\int_0^\infty ru\, dr}{\bar{u}}, \tag{10}$$

the mean specific gravity difference

$$\left(\overline{\frac{\Delta\gamma_0}{\gamma_0}}\right) = \frac{2}{\bar{b}^2}\int_0^\infty r\frac{\Delta\gamma_0}{\gamma_0}dr, \tag{11}$$

and the shape factor

$$I = \frac{2\int_0^\infty ru\left(\frac{\Delta\gamma_0}{\gamma_0}\right)dr}{\bar{u}\left(\overline{\frac{\Delta\gamma_0}{\gamma_0}}\right)\bar{b}^2} = \frac{\int_0^\infty ru\left(\frac{\Delta\gamma_0}{\gamma_0}\right)dr\int_0^\infty ru\,dr}{\int_0^\infty r\left(\frac{\Delta\gamma_0}{\gamma_0}\right)dr\int_0^\infty ru^2dr}. \tag{12}$$

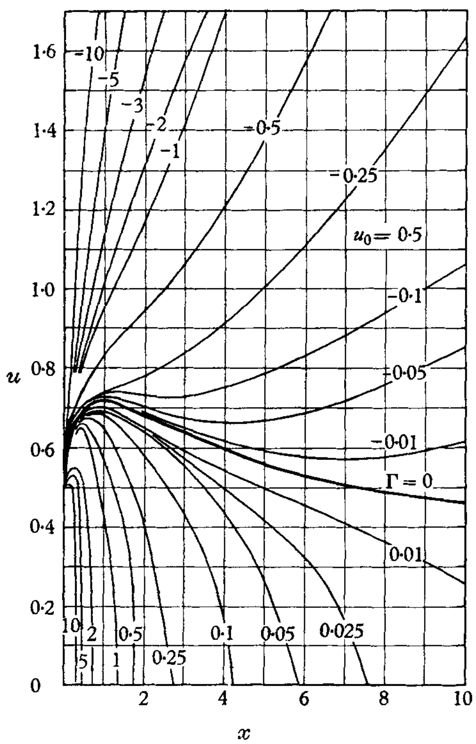

FIGURE 5. The variation of plume velocity u with height x and atmospheric lapse rate Γ. Parameter u_0 is initial plume velocity.

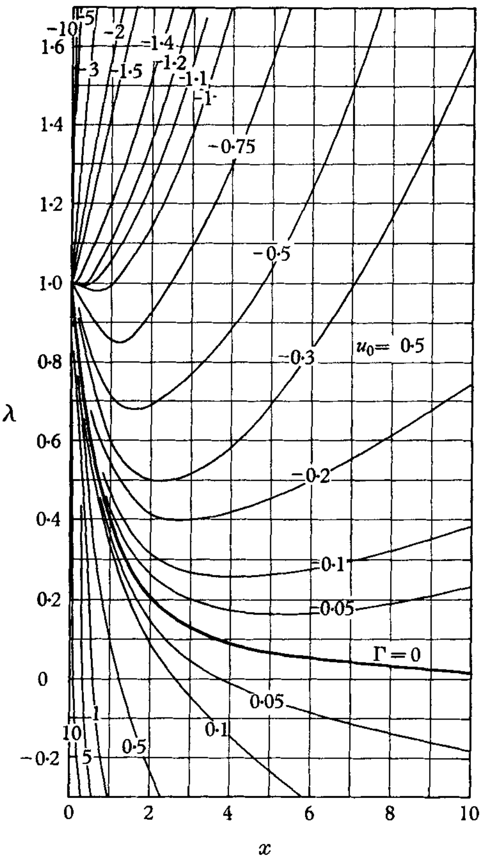

FIGURE 6. The variation of plume buoyancy λ with height x and atmospheric lapse rate Γ. Parameter u_0 is initial plume velocity.

The factor I depends upon the profile shapes as can be seen from the second form. In particular, if the velocity and specific gravity difference are of the same shape

$$u = Af(r), \quad \frac{\Delta\gamma_0}{\gamma_0} = Bf(r), \quad \text{and hence} \quad I = 1. \tag{13}$$

Furthermore, if

$$f(r) = e^{-r^2/\beta^2},$$

as is sometimes assumed, then

$$\bar{u} = \tfrac{1}{2}\bar{u}_{\max.}, \quad \left(\overline{\frac{\Delta\gamma_0}{\gamma_0}}\right) = \frac{1}{2}\left(\frac{\Delta\gamma_0}{\gamma_0}\right)_{\max.}, \quad \bar{b} = \beta\sqrt{2}. \tag{14}$$

In the following neither (13) nor (14) will be assumed, but we will assume that I is independent of the height $\bar{x}$. If this is not so, then either a two- (or three-) dimensional theory is essential, or some assumption concerning the variation of I with $\bar{x}$ must be introduced.

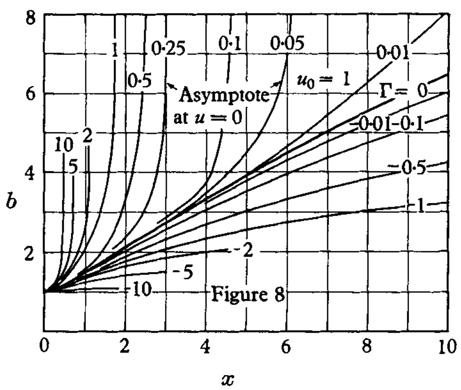

Figure 7. The variation of plume width b with height x and atmospheric lapse rate Γ. Parameter u_0 is initial plume velocity.

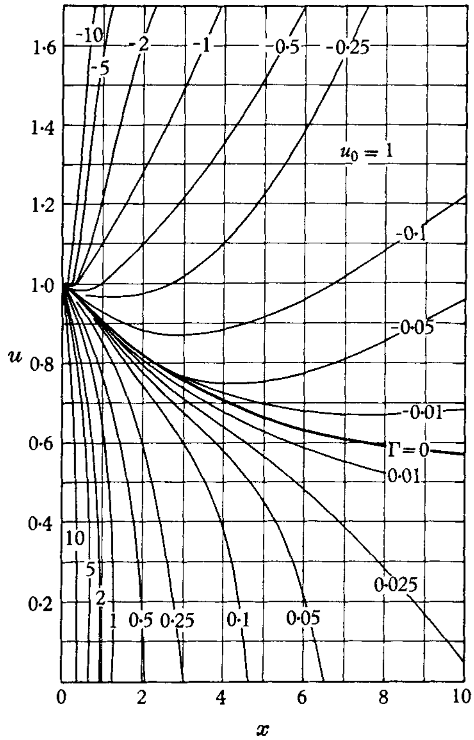

FIGURE 8. The variation of plume velocity u with height x and atmospheric lapse rate Γ. Parameter u_0 is initial plume velocity.

In terms of the above mean values, the three equations (6), (7), (8) become

$$\frac{d}{d\bar{x}}\bar{u}\bar{b}^2 = \alpha\bar{u}\bar{b}, \tag{15}$$

$$\frac{d}{d\bar{x}}\bar{u}^2\bar{b}^2 = g\bar{b}^2\frac{\overline{\Delta\gamma_0}}{\gamma_0}, \tag{16}$$

$$\frac{d}{d\bar{x}}\frac{\bar{u}\overline{\Delta\gamma_0}\bar{b}^2}{\gamma_0} = -\frac{\bar{u}\bar{b}^2}{I}\frac{d\ln T_{\infty 0}}{d\bar{x}}, \tag{17}$$

where $\alpha\bar{u}\bar{b} = \lim_{r\to\infty}(-2rv)$ is the mass entrainment rate and α is supposed to be a constant. At the level of the ground (using the atmospheric pressure at the ground as p_0), the heat flux Q is given by

$$Q = 2\pi c_p T_\infty \int_0^\infty \frac{r u \Delta\gamma_0}{\gamma_0}\,dr = \pi c_p T_\infty \bar{u}\left(\frac{\overline{\Delta\gamma_0}}{\gamma_0}\right)\bar{b}^2, \tag{18}$$

where the last expression assumes a top-hat distribution.

It has been customary to define dimensionless variables by use of Q. This has advantages when dealing with a point source of energy, but for the present general problem of arbitrary initial size, velocity and energy, the following is somewhat better since it simplifies the treatment of variable atmospheres. Let

$$\left.\begin{aligned} x &= \frac{\alpha\bar{x}}{\bar{b}_0}, \quad b = \frac{\bar{b}}{\bar{b}_0}, \quad u = \left(\frac{\alpha}{(\overline{\Delta\gamma_0/\gamma_0})_0}\right)^{\frac{1}{2}} \frac{\bar{u}}{(\bar{b}_0 g)^{\frac{1}{2}}}, \\ \lambda &= \frac{\overline{\Delta\gamma_0/\gamma_0}}{(\overline{\Delta\gamma_0/\gamma_0})_0}, \quad \Gamma = \frac{\bar{b}_0}{\alpha I(\overline{\Delta\gamma_0/\gamma_0})_0}\frac{d\ln T_{\infty 0}}{d\bar{x}} \end{aligned}\right\} \tag{19}$$

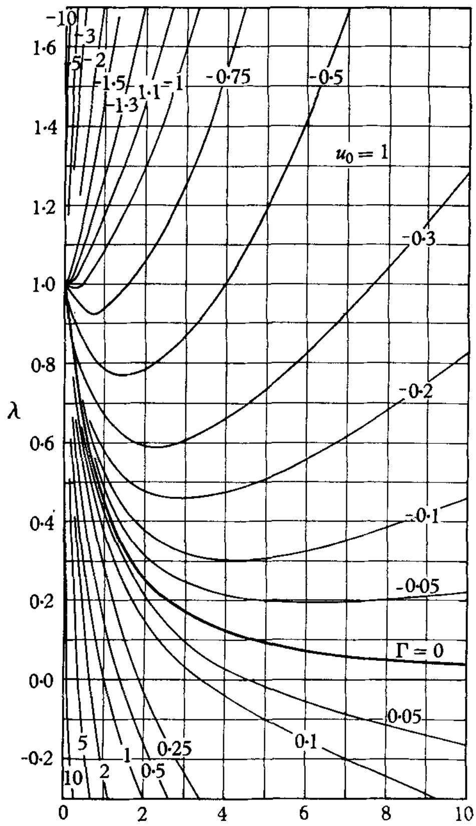

FIGURE 9. The variation of plume buoyancy λ with height x and atmospheric lapse rate Γ. Parameter u_0 is initial plume velocity.

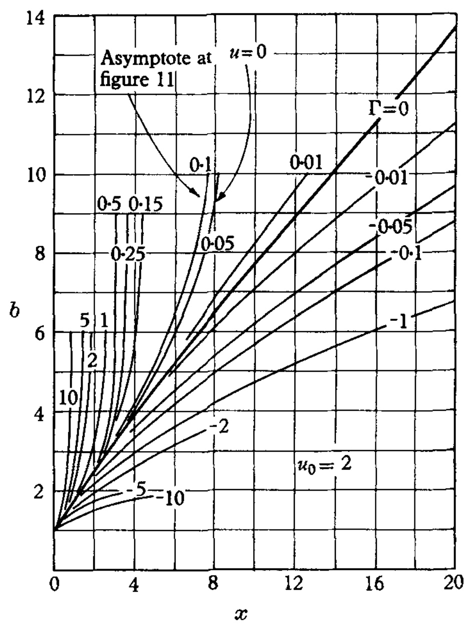

FIGURE 10. The variation of plume width b with height x and atmospheric lapse rate Γ. Parameter u_0 is initial plume velocity.

(Γ is the dimensionless lapse rate). The subscript 0 outside the brackets stand for the initial value of the potential variable. All quantities marked with an overbar have physical dimensions. (The defined quantities should not be confused with those of equations (1–11).) Note that u is the Froude number based upon buoyancy (except for α).

The final form of the conservation equations now is

$$\frac{d}{dx}ub^2 = ub, \tag{20}$$

$$\frac{d}{dx}u^2b^2 = \lambda b^2, \tag{21}$$

$$\frac{d}{dx}\lambda ub^2 = -\Gamma ub^2. \tag{22}$$

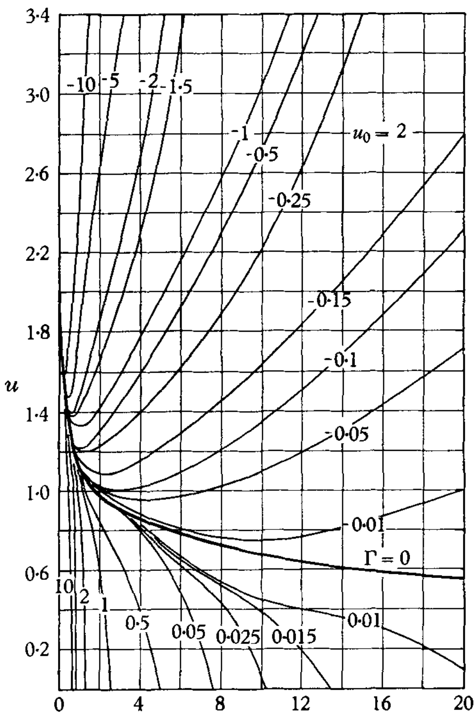

FIGURE 11. The variation of plume velocity u with height x and atmospheric lapse rate Γ. Parameter u_0 is initial plume velocity.

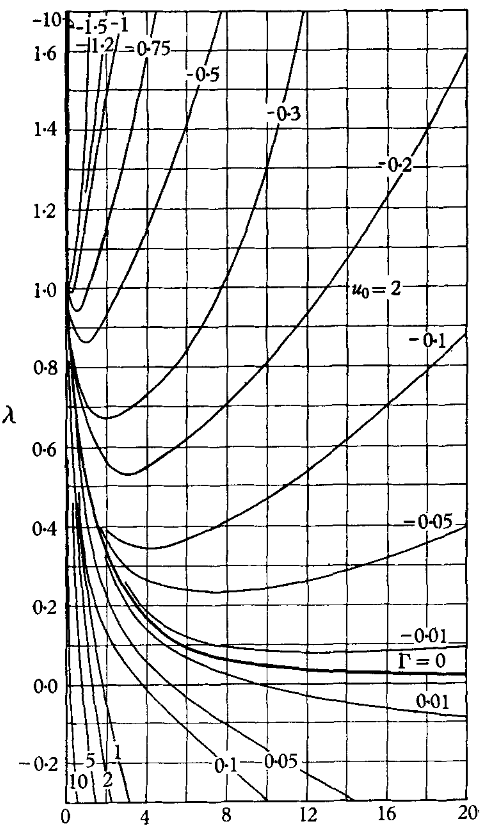

FIGURE 12. The variation of plume buoyancy λ with height x and atmospheric lapse rate Γ. Parameter u_0 is initial plume velocity.

If we are interested in an arbitrary atmosphere, the lapse rate is a function of height ($\Gamma = \Gamma(x)$). In view of the approximations already made it appears sufficiently accurate to take Γ as constant over ranges of x. Thus for any range (n) the boundary conditions at its lower edge ($x_n = 0$) are simply related to the solution at the top ($x_{n-1} = L_{n-1}$) of the next lower range ($n-1$). Thus for any range of height, the boundary conditions are

$$x_n = 0, \quad b_n = 1, \quad \lambda_n = 1,$$

$$u_n = u_0 \text{ for new range} = \left[\frac{u_{n-1}}{(\lambda_{n-1}b_{n-1})^{\frac{1}{2}}}\right]_{x_{n-1}=L_{n-1}} \tag{23}$$

The solutions to equations (20), (21), (22), with boundary conditions (23) were drawn by a Pace computer for the following values of the two parameters

$$u_0 = 0{\cdot}2,\ 0{\cdot}5,\ 1,\ 2,\ 5, \quad -10 < \Gamma < 10.$$

These values were chosen to cover the range of fire convection columns. The value $u_0 = 0$ was omitted in spite of the fact that a hot plate appears to initiate such a rising column. The equations yield an unreal singular solution (see (24) below). This is as it should be, since our equations neglect vertical conduction.

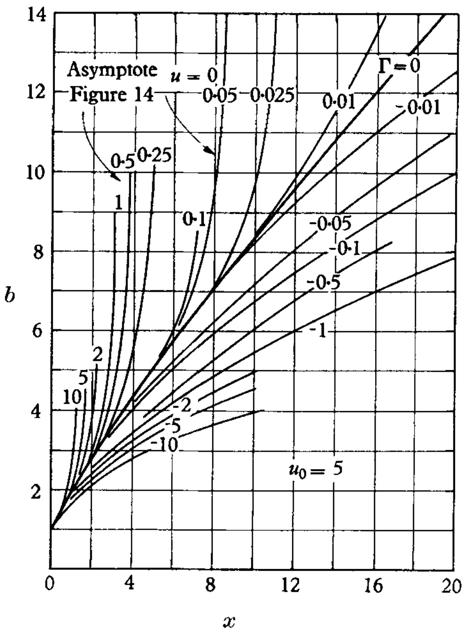

FIGURE 13. The variation of plume width b with height x and atmospheric lapse rate Γ. Parameter u_0 is initial plume velocity.

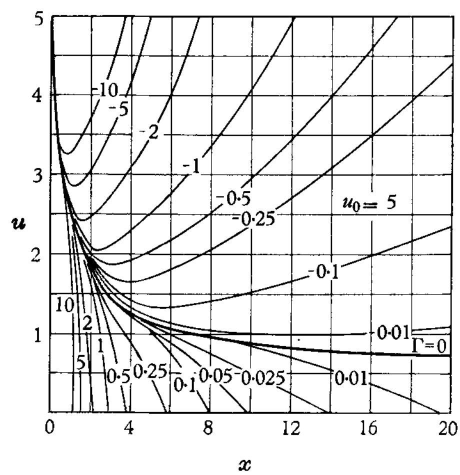

Figure 14. The variation of plume velocity u with height x at atmospheric lapse rate Γ. Parameter u_0 is initial plume velocity.

Figures 1 to 15 present velocity u, density defect λ and column width b as functions of x and the parameter Γ for each of the five chosen values of initial velocity u_0.

As mentioned before, the solution may be developed from these curves by interpolation in Γ at constant u_0 until such atmospheric height that the $d\,(\ln T_{\infty 0})/dx$ changes to a new constant value. At this height the computation must be started again using new values of u_0 and Γ (and, of course, a new elevation datum $x = 0$).

At the start of the computation, or when changing from one constant region to another, the value of u_0 may not fall on one of the values for which curves are presented in figures 1 to 15. One can sometimes with sufficient accuracy interpolate between available u_0 as well as Γ values. Such double interpolation can be avoided by calculating a short range of altitude by the following approximate formulas:

$$\left.\begin{aligned} u &= u_0+\left(\frac{1}{u_0}-u_0\right)x-\frac{x^2}{2u_0^3}\left(1+\frac{2\Gamma+3}{2}u_0^2\right)+\ldots,\\ \lambda &= 1-(\Gamma+1)x+\frac{x^2}{4}\left(2\Gamma-\frac{1}{2u_0^2}\right)+\ldots,\\ b &= 1+\left(1-\frac{1}{2u_0^2}\right)x+\frac{1}{8u_0^4}\{3+2u_0^2(\Gamma-2)\}x^2+\ldots.\end{aligned}\right\} \tag{24}$$

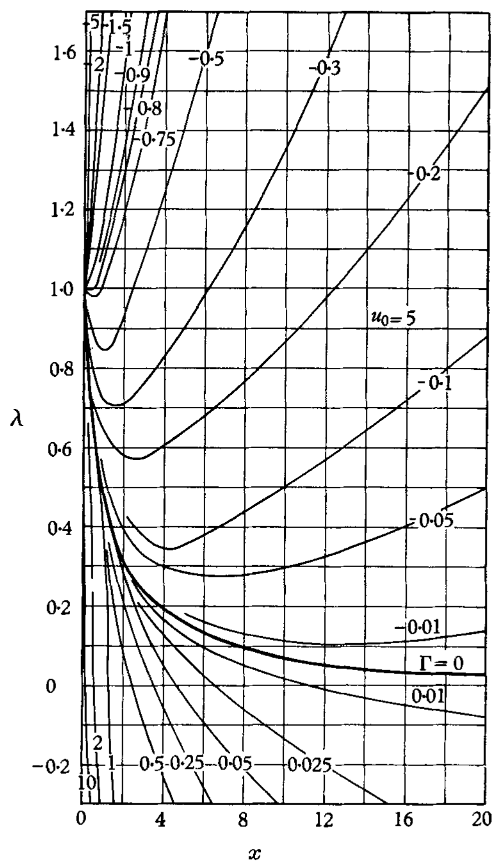

FIGURE 15. The variation of plume buoyancy λ with height x and atmospheric lapse rate Γ. Parameter u_0 is initial plume velocity.

This calculation should be continued to such a value of x that if a new range is again started, u_0 has one of the 5 available values. This value of x, given to the first order in x, is

$$x = \frac{4u_0(u_0^*-u_0)}{5+2u_0^2(\Gamma-2)}, \tag{25}$$

where u_0, Γ are values appropriate to the short range being computed and u_0^* is the nearest u_0 for one of the available solution graphs.

To illustrate the method of use of the solutions herein presented, the following problem is solved. Suppose a fire of 10 m diameter releases heat at the rate

10^8 cal/sec. Suppose, in agreement with some rough measurements, the upward velocity just above the flames is $\bar{u} = 3$ m/sec.

Atmospheric conditions chosen are those which existed over Seattle, Washington on 1 August 1957. The temperature, potential temperature and piecewise constant lapse rate are shown in figure 16.

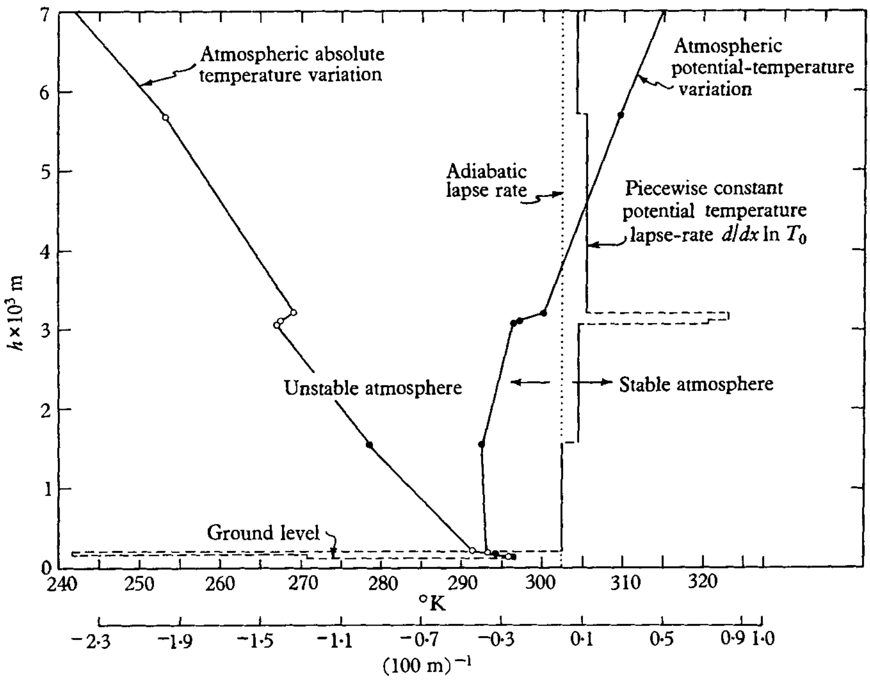

FIGURE 16. Variation of temperature, potential temperature and potential temperature lapse-rate with height above sea level on 1 August 1957, Seattle, Washington.

By (18) and the given data, one finds that at the ground level ($h = 125$ m above sea level) the buoyancy value is

$$\frac{\Delta\gamma_{00}}{\gamma_{00}} = 0\cdot614,$$

and, by (19), using $\alpha = 0\cdot1$, that $u_0 = 0\cdot13$, $\Gamma = -0\cdot021$. This completes the first line of table 1. The value $u_0 = 0\cdot13$ does not correspond to any of the available solution curves. The nearest is $u_0 = 0\cdot2$. Equation (25*a*) indicates that, at $x = 0\cdot007$ diameters, a new start would result in $u_0 = 0\cdot2$. Now equations (24) give line 2 of table 1 while line 3 gives the starting values for use with the solution curves.

The height of the top of the first lapse-rate range is 52 m. Thus $\bar{x}$ varies from 0 to 51·3 m, or in dimensionless form, from 0 to 64. The solution curves now give values for u, b, λ at $x = 0\cdot64$, $u_0 = 0\cdot2$, $\Gamma = -0\cdot017$. Thus line 4 of table 1 can be completed.

The next lapse-rate range begins at $h = 52$ m and ends at 75 m. This range is computed as above except that interpolation between $u_0 = 1$ and $u_0 = 2$ is more convenient than use of (25). Additional lapse-rate ranges are computed in the same manner up to range 5 where the velocity falls to zero. Thus we find that at $h = 2180$ m the column stops rising. The buoyancy falls essentially to zero at a height of 704 m and goes negative at a height of about 1400.

The resultant changes in the column velocity, diameter and buoyancy are plotted in figure 17.

h m above ground	Range	$\bar{x}$ m	x initial range diam.	Γ	u range dimensionless	u initial dimensionless	$\bar{u}$ in m/sec	b initial range diam.	b fire diam.	$\bar{b}$ m	λ fraction of initial range value	λ fraction of initial fire value	λ local density difference ratio
0	1	0	0	−0·021	0·13	0·13	3·0	1	1	10	1	1	0·614
0·7	1	0·7	0·007	−0·021	0·183	0·183	4·2	0·80	0·8	8	0·99	0·99	0·608
0·7	2	—	0	−0·017	0·20	—	—	1	0·8	—	1	—	—
52	2	51·3	0·64	−0·017	0·63	0·573	13·1	0·80	0·64	6·4	0·45	0·446	0·274
52	3	—	0	−0·057	1·07	—	—	1	0·64	—	1	0·446	—
75	3	23	0·36	−0·057	1·05	0·561	12·9	1·12	0·72	7·2	0·7	0·31	0·190
75	4	—	0	~0	1·186	—	—	1	—	—	1	—	—
579	4	504	7	~0	0·636	0·301	7·0	5·16	3·72	37·2	0·062	0·028	0·017
579	5	—	0	—	1·132	—	—	1	—	—	1	—	—
951	5	372	1	0·021	1·132	0·251	5·8	1·51	5·62	56·2	0·43	0·020	0·012
1395	5	816	2·19	0·021	0·808	0·215	5·0	2·29	8·54	85·4	0·221	0·0042	0·0025
1395	6	—	—	—	1·164	—	—	1	—	—	1	—	—
1600	6	205	0·24	2·76	1·164	0·18	4·2	2·55	9·5	95·0	—	−0·04	−0·025
2180	6	785	0·92	—	0	0	0	∞	∞	∞	—	−0·3	−0·18

TABLE 1

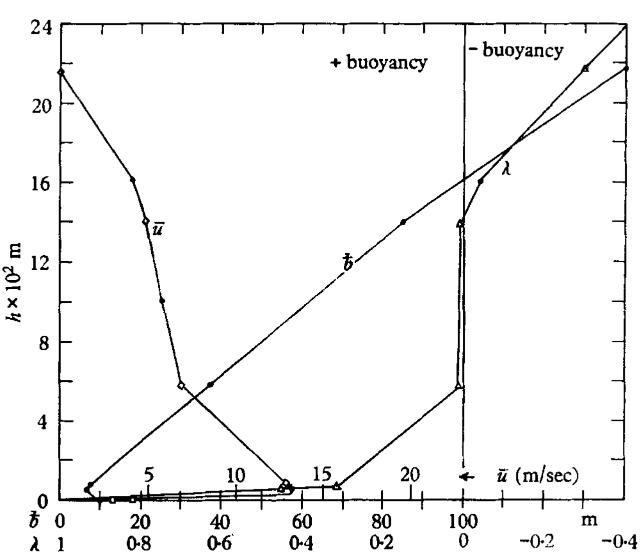

FIGURE 17. Properties of buoyancy plume in atmosphere of figure 16 due to 10 m diameter fire liberating 10^5 k cal/sec energy. Initial values: $\bar{b} = 10$ m, $\bar{u} = 3$ m/sec, $(\Delta\gamma_0/\gamma_0)_0\, g = 0{\cdot}602$.

Appendix

If terms are not neglected, equations (6), (7), (8), are replaced by

$$\frac{\partial}{\partial \bar{x}}\rho_0 ur+\frac{\partial}{\partial r}\rho_0 vr-\rho_0\frac{ru}{kp}\rho_{\infty 0}g\left(\frac{p}{p_0}\right)^{1/k} = 0, \tag{1a}$$

$$r\left(u\frac{\partial u}{\partial \bar{x}}+v\frac{\partial u}{\partial r}\right) = r\frac{\Delta\gamma_0}{\gamma_0}g+\frac{1}{\rho_0}\frac{\partial}{\partial r}r\tau_0, \tag{2a}$$

$$r\left(u\frac{\partial T_0}{\partial \bar{x}}+v\frac{\partial T_0}{\partial r}\right) = \frac{ru\rho_{\infty 0}g}{p}\left(\frac{p}{p_0}\right)^{1/k}\left\{T_0\frac{k-1}{k}-\frac{p_0}{\rho_0 c_p}\right\}-\frac{1}{\rho_0 c_p}\frac{\partial}{\partial r}(rq_0). \tag{3a}$$

Introducing the ideal gas relation between T_0 and $\Delta\gamma_0/\gamma_0$,

$$T_0 = T_{\infty 0}\frac{\Delta\gamma_0}{\gamma_0}+T_{\infty 0},$$

equation $3a$ becomes

$$r\left(u\frac{\partial}{\partial \bar{x}}\frac{\Delta\gamma_0}{\gamma_0}+v\frac{\partial}{\partial r}\frac{\Delta\gamma_0}{\gamma_0}\right)+ur\frac{d}{dx}\ln T_{\infty 0} = -ru\frac{\Delta\gamma_0}{\gamma_0}\frac{d}{dx}\ln T_{\infty 0}$$

$$+ru\rho_{\infty 0}g\left(\frac{p_0}{p}\right)^{(k-1)/k}\left[\frac{k-1}{k}\frac{\rho_{\infty 0}}{\rho_0}-\frac{1}{\rho_0 c_p T_{\infty 0}}\right]-\frac{1}{\rho_0 c_p T_{\infty 0}}\frac{\partial}{\partial r}(rq_0). \tag{4a}$$

Following the steps of derivation and definition of mean and dimensionless quantities as in the paper, assuming ρ_0 constant in $1a$, and further noting that

$$\frac{k-1}{k} = \frac{R}{c_p} = \frac{p_0}{c_p T_0 \rho_0},$$

R being the gas constant per unit mass, we find that equations (20), (21), (22) become

$$\frac{d}{dx}b^2u = bu-b^2ux^*,$$

$$\frac{d}{dx}b^2u^2 = b^2\lambda-b^2u^2x^*,$$

$$\frac{d}{dx}b^2u\lambda = -b^2u\Gamma-b^2u\lambda\,(\delta^*-x^*),$$

where

$$\delta^* = \frac{b_0}{\alpha}\frac{d}{d\bar{x}}\ln T_{\infty 0} \quad\text{and}\quad x^* = \frac{b_0}{\alpha}\frac{\rho_{\infty 0}}{kp}g\left(\frac{p}{p_0}\right)^{1/k} = \frac{b_0 g}{\alpha kRT_{\infty 0}\left(\frac{p}{p_0}\right)^{(k-1)/k}}.$$

x^* is a measure of the variation of the potential density with height and in view of the equation (1) is given as an explicit function of $(p_0/p)^{(k-1)/k}$. x^* does not become important unless we are dealing with large fires and large heights (large pressure ratios). For example, for a fire size of $10k$ and for $(p_0/p) \approx 4, x^* = 0{\cdot}3$. This pressure ratio corresponds almost to the top of the troposphere. For most fires and an atmosphere of several stable and unstable lapse rates, this height will never be reached and x^* will therefore be even less important. δ^* can be neglected compared with Γ, as is apparent from their definition, for temperatures $T_0 \ll 2T_{\infty 0}$.

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